Real Options and Interactions With Financial Flexibility

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■ Many academics and practicing managers now recognize that the net present value (NPV) rule and other discounted cash flow (DCF) approaches to capital budgeting are inadequate in that they cannot properly capture management's flexibility to adapt and revise later decisions in response to unexpected market developments. Traditional NPV makes implicit assumptions concerning an "expected scenario" of cash flows and presumes management's passive commitment to a certain "operating strategy" (e.g., to initiate the project immediately, and operate it continuously at base scale until the end of its prespecified expected useful life).

In the actual marketplace, characterized by change, uncertainty and competitive interactions, however, the realization of cash flows will probably differ from what management expected initially. As new information arrives and uncertainty about market conditions and future cash flows is gradually resolved, management may have valuable flexibility to alter its operating strategy in order to capitalize on favorable future opportunities or mitigate losses. For example, management may be able to defer, expand, contract, abandon, or otherwise alter a project at different stages during its useful operating life.

Management's flexibility to adapt its future actions in response to altered future market conditions expands an investment opportunity's value by improving its upside potential while limiting downside losses relative to management's initial expectations under passive management. The resulting asymmetry caused by managerial adaptability calls for an "expanded NPV" rule reflecting both value components: the traditional (static or passive) NPV of direct cash flows, and the option value of operating and strategic adaptability. This does not mean that tradi-

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tional NPV should be scrapped, but rather should be seen as a crucial and necessary input to an options-based, expanded NPV analysis, i.e.,

$$\text{Expanded (strategic) NPV} = \text{static (passive) NPV of expected cash flows} + \text{value of options from active management.} \quad (1)$$

An options approach to capital budgeting has the potential to conceptualize and even quantify the value of options from active management. This value is manifest as a collection of real (call or put) options embedded in capital investment opportunities, having as an underlying asset the gross project value of expected operating cash flows. Many of these real options occur naturally (e.g., to defer, contract, shut down or abandon), while others may be planned and built-in at some extra cost (e.g., to expand capacity or build growth options, to default when investment is staged sequentially, or to switch between alternative inputs or outputs). Exhibit 1 describes briefly the most common categories of encountered real options, the types of industries they are important in, and lists representative authors that have analyzed them.[1] A more comprehensive review of the real options literature is given in the first section.

This paper has two main goals. First, it provides a comprehensive overview of the existing real options literature and applications, and presents practical principles for quantifying the value of various real options. Second, it takes a first step towards extending the real options literature to recognize interactions with financial flexibility. The comprehensive literature review traces the evolution of the real options revolution, organized around thematic developments covering the early criticisms, conceptual approaches, foundations and building blocks, risk-neutral valuation and risk adjustment, analytic contributions in valuing different options separately, option interactions, numerical techniques, competition and strategic options, various applications, and future research directions. An example is then used to conceptually discuss the basic nature of the various real options that may be embedded in capital investments. Initially assuming all-equity financing, the paper presents principles useful for valuing both upside-potential operating options, such as to defer an investment or expand production, as well as various downside-protection options, such as to abandon for salvage value or switch use (inputs/outputs), and abandon project construction by defaulting on planned, staged future outlays.

Building on the above principles, the paper subsequently extends the analysis in the presence of financial leverage within a venture capital context and examines the improvement in equityholders' value as a result of additional financial flexibility, noting potential interactions with operating flexibility. The beneficial impact of staging venture capital financing in installments (thereby creating an option to abandon by the lender, as well as an option to revalue later at potentially better terms by each party), and other issues related to the mix of debt and equity venture capital financing are also explored.

The paper is organized as follows. Following the comprehensive literature review in Section I, Section II uses an example to motivate discussion of various real options and presents practical principles for valuing several such options. Section III then illustrates how options valuation can be extended to capture interactions with financial flexibility. The last section concludes and discusses some extensions.

I. A Review of the Real Options Literature

Corporate value creation and competitive position in different markets are critically determined by corporate resource allocation and the evaluation of investment opportunities. The field of capital budgeting remained stagnant for several decades, until recent developments in real options provided the tools and unlocked the possibilities to revolutionize the field. In what follows, I will attempt to describe briefly some stages in the development and evolution of the real options literature, while organizing the presentation around several broad themes. This is not an easy task, and I apologize to those authors and readers who may find my treatment here rather subjective and non-exhaustive.

A. Symptoms, Diagnosis, and Traditional Medicine: Early Critics, the Underinvestment Problem, and Alternative Valuation Paradigms

The real options revolution arose in part as a response to the dissatisfaction of corporate practitioners, strategists, and some academics with traditional capital budgeting techniques. Well before the development of real options, corporate managers and strategists were grappling intuitively with the elusive elements of managerial operating flexibility and strategic interactions. Early critics (e.g., Dean [29], Hayes and Abernathy [35], and Hayes and

[1]Parts of Exhibit 1 are adapted from Baldwin and Trigeorgis [8].

Exhibit 1. Common Real Options

Category	Description	Important In	Analyzed By
Option to defer	Management holds a lease on (or an option to buy) valuable land or resources. It can wait (*x* years) to see if output prices justify constructing a building or plant, or developing a field.	All natural resource extraction industries; real estate development; farming; paper products.	Tourinho [98]; Titman [97]; McDonald & Siegel [76]; Paddock, Siegel & Smith [83]; Ingersoll & Ross [44].
Time to build option (staged investment)	Staging investment as a series of outlays creates the option to abandon the enterprise in midstream if new information is unfavorable. Each stage can be viewed as an option on the value of subsequent stages, and valued as a compound option.	All R&D intensive industries, especially pharmaceuticals; long-development capital-intensive projects, e.g., large-scale construction or energy-generating plants; start-up ventures.	Majd & Pindyck [68]; Carr [22]; Trigeorgis [106].
Option to alter operating scale (e.g., to expand; to contract; to shut down and restart)	If market conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are *less* favorable than expected, it can reduce the scale of operations. In extreme cases, production may temporarily halt and start up again.	Natural resource industries such as mine operations; facilities planning and construction in cyclical industries; fashion apparel; consumer goods; commercial real estate.	Brennan & Schwartz [19]; McDonald & Siegel [75]; Trigeorgis & Mason [110]; Pindyck [84].
Option to abandon	If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets in secondhand markets.	Capital intensive industries, such as airlines and railroads; financial services; new product introductions in uncertain markets.	Myers & Majd [82].
Option to switch (e.g., outputs or inputs)	If prices or demand change, management can change the output mix of the facility ("product" flexibility). Alternatively, the same outputs can be produced using different types of inputs ("process" flexibility).	*Output shifts:* any good sought in small batches or subject to volatile demand, e.g., consumer electronics; toys; specialty paper; machine parts; autos. *Input shifts:* all feedstock-dependent facilities, e.g., oil; electric power; chemicals; crop switching; sourcing.	Margrabe [69]; Kensinger [50]; Kulatilaka [55]; Kulatilaka & Trigeorgis [63].
Growth options	An early investment (e.g., R&D, lease on undeveloped land or oil reserves, strategic acquisition, information network/infrastructure) is a prerequisite or link in a chain of interrelated projects, opening up future growth opportunities (e.g., new generation product or process, oil reserves, access to new market, strengthening of core capabilities). Like interproject compound options.	All infrastructure-based or strategic industries, especially high-tech, R&D, or industries with multiple product generations or applications (e.g., computers, pharmaceuticals); multinational operations; strategic acquisitions.	Myers [80]; Brealey & Myers [16]; Kester [51], [52]; Trigeorgis [100]; Pindyck [84]; Chung & Charoenwong [23].
Multiple interacting options	Real-life projects often involve a "collection" of various options, both upward-potential enhancing calls and downward-protection put options present in combination. Their combined option value may differ from the sum of separate option values, i.e., they interact. They may also interact with financial flexibility options.	Real-life projects in most industries discussed above.	Brennan & Schwartz [19]; Trigeorgis [106]; Kulatilaka [58].

Garvin [36]) recognized that standard discounted cash flow (DCF) criteria often undervalued investment opportunities, leading to myopic decisions, underinvestment and eventual loss of competitive position, because they either ignored or did not properly value important strategic considerations. Decision scientists further maintained that the problem lay in the application of the wrong valuation techniques altogether, proposing instead the use of simulation and decision tree analysis (see Hertz [38] and Magee [67]) to capture the value of future operating flexibility associated with many projects. Proponents (e.g., Hodder and Riggs [41] and Hodder [40]) have argued that the problem arises from misuse of DCF techniques as commonly applied in practice. Myers [81], while confirming that part of the problem results from various misapplications of the underlying theory, acknowledges that traditional DCF methods have inherent limitations when it comes to valuing investments with significant operating or strategic options (e.g., in capturing the sequential interdependence among investments over time), suggesting that option pricing holds the best promise of valuing such investments. Later, Trigeorgis and Mason [110] explain that option valuation can be seen operationally as a special, economically corrected version of decision tree analysis that is better suited in valuing a variety of corporate operating and strategic options, while Teisberg [95] provides a practical comparative discussion of the DCF, decision analysis, and real option valuation paradigms. Baldwin and Clark [5] discuss the importance of organizational capabilities in strategic capital investment, while Baldwin and Trigeorgis [8] propose remedying the underinvestment problem and restoring competitiveness by developing specific adaptive capabilities viewed as an infrastructure for acquiring and managing real options.

B. A New Direction: Conceptual Real Options Approaches

Building on Myers' [80] initial idea of thinking of discretionary investment opportunities as "growth options," Kester [51] conceptually discusses strategic and competitive aspects of growth opportunities. Other general, conceptual real options frameworks are presented in Mason and Merton [71], Trigeorgis and Mason [110], Trigeorgis [100], Brealey and Myers [16], and Kulatilaka and Marcus [59], [60]. Mason and Merton [71], for example, provide a good discussion of many operating as well as financing options, and integrate them in a project financing for a hypothetical, large-scale energy project.

C. Generic Medicine: Foundations and Building Blocks

The quantitative origins of real options, of course, derive from the seminal work of Black and Scholes [13] and Merton [78] in pricing financial options. Cox, Ross, and Rubinstein's [27] binomial approach enabled a more simplified valuation of options in discrete-time. Margrabe [69] values an option to exchange one risky asset for another, while Stulz [94] analyzes options on the maximum (or minimum) of two risky assets and Johnson [45] extends it to several risky assets. These papers have the potential to help analyze the generic option to switch among alternative uses and related options (e.g., abandon for salvage value or switch among alternative inputs or outputs). Geske [31] values a compound option (i.e., an option to acquire another option), which, in principle, may be applied in valuing growth opportunities which become available only if earlier investments are undertaken. Carr [22] combines the above two building blocks to value sequential (compound) exchange options, involving an option to acquire a subsequent option to exchange the underlying asset for another risky alternative. Kulatilaka [55] and [57] describes an equivalent dynamic programming formulation for the option to switch among operating modes. The above line of work has the potential, in principle, to value investments with a series of investment outlays that can be switched to alternative states of operation, and particularly to eventually help value strategic interproject dependencies.

D. Slightly Different Medicine: Risk-Neutral Valuation and Risk Adjustment

The actual valuation of options in practice has been greatly facilitated by Cox and Ross's [26] recognition that an option can be replicated (or a "synthetic option" created) from an equivalent portfolio of traded securities. Being independent of risk attitudes or capital market equilibrium considerations, such risk-neutral valuation enables present-value discounting, at the risk-free interest rate, of expected future payoffs (with actual probabilities replaced with risk-neutral ones), a fundamental characteristic of "arbitrage-free" price systems involving traded securities. Rubinstein [87] further showed that standard option pricing formulas can be alternatively derived under risk aversion, and that the existence of continuous trading opportunities enabling a riskless hedge or risk neutrality are not really necessary. Mason and Merton [71] and Kasanen and Trigeorgis [48] maintain that real options may, in principle, be valued similar to financial options, even though they may not be traded, since in capital budgeting we are interested in determining what the project cash flows would be worth if they were traded in the market, i.e., their contribution to the *market* value of a publicly traded firm. The existence of a traded "twin security" (or dynamic portfolio) that has the same risk characteristics (i.e., is perfectly correlated) with the nontraded real asset in complete markets is sufficient for real option valuation. More generally, Constantinides [24], Cox, Ingersoll, and Ross [28, lemma 4], and Harrison and Kreps [34], among others, have suggested that any contingent claim on an asset, traded or not, can be priced in a world with systematic risk by replacing its actual growth rate with a certainty-equivalent rate (by subtracting a risk premium that would be appropriate in market equilibrium), and then behaving as if the world were risk-neutral. This is analogous to discounting certainty-equivalent cash flows at the risk-free rate, rather than actual expected cash flows at a risk-adjusted rate. For traded assets in equilibrium or for those real assets with no systematic risk (e.g., R&D, exploration or drilling for certain precious metals or natural resources), the certainty-equivalent or risk-neutral rate just equals the risk-free interest rate (minus any dividends). However, if the underlying asset is not traded, as may often be the case in capital budgeting associated options, its growth rate may actually fall below the equilibrium total expected return required of an equivalent-risk traded financial security, with the difference or "rate of return shortfall" necessitating a dividend-like adjustment in option valuation (e.g., see McDonald and Siegel [74] and [75]). If the underlying asset is traded in futures markets, though, this dividend- (or convenience-yield-) like return shortfall or rate of foregone earnings can be easily derived from the futures prices of contracts with different maturities (see Brennan and Schwartz [19]). In other cases, however, estimating this return shortfall may require use of a market equilibrium model (e.g., see McDonald and Siegel [75]).

E. A Tablet for Each Case: Valuing Each Different Real Option Separately

There came a series of papers which gave a boost to the real options literature by focusing on valuing quantitatively — in many cases, deriving analytic, closed-form solutions— one type after another of a variety of real options, although each option was typically analyzed in isolation. As summarized in Exhibit 1, the option to defer or initiate investment has been examined by McDonald and Siegel [76], by Paddock, Siegel, and Smith [83] in valuing offshore petroleum leases, and by Tourinho [98] in valuing reserves of natural resources. Ingersoll and Ross [44] reconsider the decision to wait in light of the beneficial impact of a potential future interest rate decline on project value. Majd and Pindyck [68] value the option to delay sequential construction for projects that take time to build, or there is a maximum rate at which investment can proceed. Carr [22] and Trigeorgis [106] also deal with valuing sequential or staged (compound) investments. Trigeorgis and Mason [110] and Pindyck [84] examine options to alter (i.e., expand or contract) operating scale or capacity choice. The option to temporarily shut down and restart operations was analyzed by McDonald and Siegel [75] and by Brennan and Schwartz [19]. Myers and Majd [82] analyze the option to permanently abandon a project for its salvage value seen as an American put option. Options to switch use (i.e., outputs or inputs) have been examined, among others, by Margrabe [69], Kensinger [50], Kulatilaka [55], and Kulatilaka and Trigeorgis [63]. Baldwin and Ruback [7] show that future price uncertainty creates a valuable switching option that benefits short-lived projects. Future investment opportunities that are seen as corporate growth options are discussed in Myers [80], Brealey and Myers [16], Kester [51] and [52], Trigeorgis and Mason [110], Trigeorgis [100], Pindyck [84], and Chung and Charoenwong [23].

F. The Tablets Interact: Multiple Options and Interdependencies

Despite its enormous theoretical contribution, the focus of the earlier literature on valuing individual real options (i.e., one type of option at a time) has nevertheless limited its practical value. Real-life projects are often more complex in that they involve a collection of multiple real options whose values may interact. An early exception is Brennan and Schwartz [19], who determine the combined value of the options to shut down (and restart) a mine, and to abandon it for salvage. They recognize that partial irreversibility resulting from the costs of switching the mine's operating state may create a persistence, inertia or *hysteresis* effect, making it long-term optimal to remain in the same operating state even though short-term considerations (i.e., current cash flows) may seem to favor immediate switching. Although hysteresis can be seen as a form of interaction between early and later decisions, Brennan and Schwartz do not explicitly address the interactions among individual option values. Trigeorgis [106] focuses on the nature of real option interactions, pointing out, for example, that the presence of subsequent options can increase the value of the effective underlying asset for earlier options, while exercise of prior real options may alter (e.g., expand or contract) the underlying asset itself, and hence the value of subsequent options on it. Thus, the combined value of a collection of real options may differ from the sum of separate option values. Using a numerical analysis method suitable for valuing complex multi-option investments (Trigeorgis [104]), he presents the valuation of options to defer, abandon, contract or expand investment, and switch use in the context of a generic investment, first with each option in isolation and later in combination. He shows, for example, that the incremental value of an additional option, in the presence of other options, is generally less than its value in isolation and declines as more options are present. More generally, he identifies situations where option interactions can be small or large and negative as well as positive. Kulatilaka [58] subsequently examines the impact of interactions among three such options on their optimal exercise schedules. The recent recognition of real option interdependencies should subsequently enable a smoother transition from a theoretical stage to an application phase.

G. The Bitter Pill: Numerical Techniques

In the more complex real-life option situations, such as those involving multiple interacting real options, analytic solutions may not exist and one may not even be always able to write down the set of partial differential equations describing the underlying stochastic processes. The ability to value such complex option situations has been enhanced, however, with various numerical analysis techniques, many of which take advantage of risk-neutral valuation. Generally, there are two types of numerical techniques for option valuation: (*i*) those that approximate the underlying stochastic processes directly and are generally more intuitive; and (*ii*) those approximating the resulting partial differential equations. The first category includes Monte Carlo simulation used by Boyle [14], and various lattice approaches such as Cox, Ross, and Rubinstein's [27] standard binomial lattice method, and Trigeorgis' [104] log-transformed binomial method; the latter are particularly well-suited to valuing complex projects with multiple embedded real options, a series of investment outlays, dividend-like effects, as well as option interactions. Boyle [15] shows how lattice frameworks can be extended to handle two state variables, while Hull and White [43] suggest a control variate technique to improve computational efficiency when a similar derivative asset with an analytic solution is available. Examples of the second category include numerical integration, and implicit or explicit finite difference schemes used by Brennan [17], Brennan and Schwartz [18], and Majd and Pindyck [68]. Finally, a number of analytic approximations are also available: Geske and Johnson [32] have proposed a compound-option analytic polynomial approximation approach; Barone-Adesi and Whaley [9] have suggested a quadratic approximation, while others have used various problem-specific heuristic approximations. A comprehensive review of such numerical techniques is given in the articles by Geske and Shastri [33] and Trigeorgis [104], as well as in a book by Hull [42].

H. The General Environment: Competition and Strategic Options

An important area that deserves more attention, and where real options have the potential to make a significant difference, is that of competition and strategy. Sustainable competitive advantages resulting from patents, proprietary technologies, ownership of valuable natural resources, managerial capital, reputation or brand name, scale, and market power, empower companies with valuable options to grow through future profitable investments and to more effectively respond to unexpected adversity or opportunities in a changing technological, competitive, or general business environment. A number of economists have addressed several competitive and strategic aspects of capital investment early on. For example, Roberts and Weitzman [86] find that in sequential decision-making, it may be worthwhile to undertake investments with negative NPV when early investment can provide information about future project benefits, especially when their uncertainty is greater. Baldwin [3] finds that optimal sequential investment for firms with market power facing irreversible decisions may require a positive premium over NPV to compensate for the loss in value of future opportunities that results from undertaking an investment. Pindyck [84] analyzes options to choose capacity under product price uncertainty when investment is, again, irreversible. Dixit [30] considers firm entry and exit decisions under uncertainty, showing that in the presence of sunk or switching costs it may not be long-term optimal to reverse a decision even when prices appear attractive in the short-term. Bell [10] combines Dixit's entry and exit decisions with Pindyck's capacity options for the multinational firm under volatile exchange rates. Kogut and Kulatilaka [53] analyze the international plant location option in the presence of mean-reverting exchange rate volatility, while Kulatilaka and

Marks [61] examine the strategic bargaining value of flexibility in the firm's negotiations with input suppliers.

From a more explicit real options perspective, a number of authors (e.g., Myers [81], Kester [51] and [52], Trigeorgis and Mason [110], Trigeorgis [100], Brealey and Myers [16], and Trigeorgis and Kasanen [109]) have initially dealt with competitive and strategic options rather conceptually. For example, Kester [51] develops qualitatively various competitive and strategic aspects of inter-project growth options, while Kester [52] proposes a planned sequential, rather than parallel, implementation of a collection of interrelated consumer products when learning results from early product introductions (e.g., about available shelf space needed for similar subsequent products) and when competitive advantage is eroding. Trigeorgis and Kasanen [109] also examine sequential project interdependencies and synergies as part of an ongoing strategic planning and control process. In this issue of *Financial Management,* Kasanen [47] also deals with the strategic problem of the interaction between current investments and future opportunities, using the rather novel concept of a spawning matrix structure (capturing the firm's ability to generate investment opportunities *across* projects through feedback effects) to determine an optimal mix of strategic and operating projects.

Trigeorgis [103] uses quantitative option pricing techniques to examine early investment that may preempt anticipated competitive entry, and to value the option to defer investment when impacted by random competitive entry (Trigeorgis [102]). Ang and Dukas [2] incorporate both competitive and asymmetric information, arguing that the time pattern of discounted cash flows also matters due to the possibility of premature project termination as a result of random competitive entry. Further departing from the common assumption of perfect competition, Kulatilaka and Perotti [62] examine how the investment decisions of a firm will influence the production decisions of competitors and the market price when early investment generates a cost advantage. In this issue, Smit and Ankum [91] combine the real options approach to investment timing with basic principles from game theory and industrial organization to explore various investment timing strategies in follow-up projects based on the reaction of competitors under different market structures. Supplementing options analysis with game theoretic tools capable of incorporating strategic competitive counteractions promises to be an important and challenging direction for future research.

I. Cure for All Kinds of Cases: A Variety of Applications

Besides theoretical developments, real option applications are currently also receiving increased attention. Real options valuation has been applied in a variety of contexts, such as in natural resource investments, land development, leasing, flexible manufacturing, government subsidies and regulation, R&D, new ventures and acquisitions, foreign investment and strategy, and elsewhere.

Early applications naturally arose in the area of *natural resource investments* due to the availability of traded resource or commodity prices, high volatilities and long durations, resulting in higher and better option value estimates. Brennan and Schwartz [19] first utilize the convenience yield derived from futures and spot prices of a commodity to value the options to shut down or abandon a mine. Paddock, Siegel, and Smith [83] value options embedded in undeveloped oil reserves and provide the first empirical evidence that option values are better than actual DCF-based bids in valuing offshore oil leases. Trigeorgis [101] values an actual minerals project considered by a major multinational company involving options to cancel during construction, expand production, and abandon for salvage. Bjerksund and Ekern [11] value a Norwegian oil field with options to defer and abandon. Mørck, Schwartz, and Stangeland [79] value forestry resources under stochastic inventories and prices. Stensland and Tjostheim [93] also discuss some applications of dynamic programming to natural resource exploration. In this volume, Laughton and Jacoby [65] examine biases in the valuation of real options and long-term decision-making when a mean-reversion price process is more appropriate, as may be the case in certain commodity projects, than the traditional Brownian motion or random walk assumption. They find that ignoring reversion would overestimate long-term uncertainty, but may over- or undervalue associated timing options. On the more applied side, Kemna [49] shares her experiences with Shell in analyzing actual cases involving the timing of developing an offshore oil field, valuing a growth option in a manufacturing venture, and the abandonment decision of a refining production unit, and discusses problem formulation and implementation issues in the process of adapting option theory in practice.

In the area of *land development*, Titman [97], Williams [111], Capozza and Sick [21], and Quigg [85B] show that the value of vacant land should reflect not only its value based on its best immediate use (e.g., from constructing a building now), but also its option value if development is delayed and the land is converted into its best alternative use in the future. It may thus pay to hold land vacant for its option value even in the presence of currently thriving real estate markets. Quigg [85A] reports empirical results indicating that option-based land valuation that incorporates the option to wait to develop land provides better approximations of actual market prices. In a different context, McLaughlin and Taggart [77] view the opportunity cost of using excess capacity as the change in the value of the firm's options caused by diverting capacity to an alternative use. In *leasing*, Copeland and Weston [25], Lee, Martin, and Senchack [66], McConnell and Schallheim [73], and Trigeorgis [105] value various operating options embedded in leasing contracts.

In the area of *flexible manufacturing*, the flexibility provided by flexible manufacturing systems, flexible production technology or other machinery having multiple uses has been analyzed from an options perspective by Kulatilaka [55], Triantis and Hodder [99], Aggarwal [1], Kulatilaka and Trigeorgis [63], and Kamrad and Ernst [46], among others. In this issue, Kulatilaka [56] values the flexibility provided by an actual dual-fuel industrial steam boiler that can switch between alternative energy inputs (natural gas and oil) as their relative prices fluctuate, and finds that the value of this flexibility far exceeds the incremental cost over a rigid, single-fuel alternative. Baldwin and Clark [6] study the flexibility created by modularity in design that connects components of a larger system through standard interfaces.

In the area of *government subsidies and regulation*, Mason and Baldwin [70] value government subsidies to large-scale energy projects as put options, while Teisberg [96] provides an option valuation analysis of investment choices by a regulated firm. In *research and development*, Kolbe, Morris, and Teisberg [54] discuss option elements embedded in R&D projects. Option elements involved in the staging of *start-up ventures* are discussed in Sahlman [88], Willner [112], and this article. Strategic *acquisitions* of other companies also often involve a number of growth, divestiture, and other flexibility options, as discussed by Smith and Triantis [102]. Other applications of options in the strategy area were discussed in Section I.H. earlier. On the empirical side, Kester [51] estimates that the value of a firm's growth options is more than half the market value of equity for many firms, even 70-80% for more volatile industries. Similarly, Pindyck [84] also suggests that growth options represent more than half of firm value if demand volatility exceeds 20%. In *foreign investment*, Baldwin [4] discusses various location, timing and staging options present when firms scan the global marketplace. Bell [10] and Kogut and Kulatilaka [53], among others, examine entry, capacity, and switching options for firms with multinational operations under exchange rate volatility. Hiraki [39] suggests that the Japanese bank-oriented corporate governance system serves as the basic infrastructure that enables companies to jointly develop corporate real options.

Various other option applications can be found in areas ranging from *shipping* (Bjerksund and Ekern [12]) to *environmental pollution and global warming* (e.g., Hendricks [37]). The potential for future applications itself seems like a growth option.

J. Other Sources and Future Research Directions

Other comprehensive treatments of real options can be found in the articles by Mason and Merton [71] and Trigeorgis and Mason [110], a monograph by Sick [89], an economics review article by Pindyck [85], as well as in a volume edited by Trigeorgis [107] and a book forthcoming from MIT Press (Trigeorgis [108]). The Spring 1987 Issue of the *Midland Corporate Finance Journal* and a 1991 Special Issue of *Managerial Finance* (Vol. 17, No. 2/3) have also been devoted to real options and capital budgeting. In the present issue of *Financial Management* (Autumn 1993), the articles by Laughton and Jacoby [65], Smit and Ankum [91], and Kasanen [47] are indicative of an active literature that is evolving in several new directions in modelling, competition and strategy, while the articles by Kemna [49] and Kulatilaka [56] represent recent attempts to implement real options valuation in actual case applications. Clearly, an increased attention to application and implementation issues is the next stage in the evolution of real options.

In addition to more actual case applications and tackling real-life implementation issues and problems, fruitful directions for future research, in both theory and practice, include:

(*i*) Focusing more on investments (such as in R&D, pilot or market tests, or excavations) that can "generate" information and learning (e.g., about the project's prospects) by extending/adjusting option pricing and risk-neutral valuation with Bayesian analysis or alternative (e.g., jump) processes.

(*ii*) Exploring in more depth endogenous competitive counteractions and a variety of competitive/market structure and strategic issues using a combination of game-theoretic industrial organization with option valuation tools.

(*iii*) Modelling better the various strategic and growth options.

(*iv*) Extending real options in an agency context recognizing that the potential (theoretical) value of real options may not be realized in practice if managers, in pursuing their own agenda (e.g., expansion or growth, rather than firm value maximization), misuse their discretion and do not follow the optimal exercise policies implicit in option valuation. This raises the need to design proper corrective incentive contracts by the firm (taking also into account asymmetric information).

(*v*) Recognizing better that real options may interact not only among themselves but with financial flexibility options as well, and understanding the resulting implications for the combined, interdependent corporate investment and financing decisions. In Section III, we take a first step toward recognizing such interactions among real and financial flexibility options.

(*vi*) On the practical side, applying real options to the valuation of flexibility in related areas, such as in competitive bidding, information technology or other platform investments, energy and R&D problems, international finance options, and so on.

(*vii*) Using real options to explain empirical phenomena that are amenable to observation or statistical testing, such as examining empirically whether managements of firms that are targets for acquisition may sometimes turn down tender offers in part due to the option to wait in anticipation of receiving better future offers.

(*viii*) Conducting more field, survey, or empirical studies to test the conformity of theoretical real option valuation and its implications with management's intuition and experience, as well as with actual price data when available.

II. Real Options: An Example and Valuation Principles

This section discusses conceptually the basic nature of different real options through a comprehensive example, and then illustrates some practical principles for valuing such options. To facilitate our discussion of the various real options that may be embedded in capital investments, consider first the following example.

A. Example: An Oil Extraction and Refinery Project

A large oil company has a one-year lease to start drilling on undeveloped land with potential oil reserves. Initiating the project may require certain exploration costs, to be followed by construction of roads and other infrastructure outlays, I_1. This would be followed by outlays for the construction of a new processing facility, I_2. Extraction can begin only after construction is completed, i.e., cash flows are generated only during the "operating stage" that follows the last outlay. During construction, if market conditions deteriorate, management can choose to forego any future planned outlays. Management may also choose to reduce the scale of operation by $c\%$, saving a portion of the last outlay, I_C, if the market is weak. The processing plant can be designed upfront such that, if oil prices turn out higher than expected, the rate of production can be enhanced by $x\%$ with a follow-up outlay of I_E. At any time, management may salvage a portion of its investment by selling the plant and equipment for their salvage value or switch them to an alternative use value, A. An associated refinery plant — which may be designed to operate with alternative sources of energy inputs — can convert crude oil into a variety of refined products. This type of project presents the following collection of real options:

(*i*) *The option to defer investment*. The lease enables management to defer investment for up to one year and benefit from the resolution of uncertainty about oil prices during this period. Management would invest I_1 (i.e., exercise its option to extract oil) *only if* oil prices increase sufficiently, but would not commit to the project, saving the planned outlays, if prices decline. Just before expiration of the lease, the value creation will be $\max(V - I_1, 0)$. The option to defer is thus analogous to an American call option on the gross present value of the completed project's expected operating cash flows, V, with the exercise price being equal to the required outlay, I_1. Since early investment implies sacrificing the option to wait, this option value loss is like an additional investment opportunity cost, justifying investment only if the value of cash benefits, V, actually exceeds the initial outlay by a substantial premium. As noted in Exhibit 1, the option to wait is particularly valuable in resource extraction industries, farming, paper products, and real estate development due to high uncertainties and long investment horizons.

(*ii*) *The option to default during construction (or the time-to-build option).* In most real-life projects, the required investment is not incurred as a single upfront outlay. The actual staging of capital investment as a series of outlays over time creates valuable options to "default" at any given stage (e.g., after exploration if the reserves or oil prices turn out very low). Thus, each stage (e.g., building necessary infrastructure) can be viewed as an option on the value of subsequent stages by incurring the installment cost outlay (e.g., I_1) required to proceed to the next stage, and can therefore be valued similar to compound options. This option is valuable in all R&D intensive industries, especially pharmaceuticals, in highly uncertain, long-development capital intensive industries, such as energy-generating plants or large-scale construction, and in venture capital.

(*iii*) *The option to expand.* If oil prices or other market conditions turn out more favorable than expected, management can actually accelerate the rate or expand the scale of production (by $x\%$) by incurring a follow-up cost outlay (I_E). This is similar to a call option to acquire an additional part ($x\%$) of the base-scale project, paying I_E as exercise price. The investment opportunity with the option to expand can be viewed as the base-scale project plus a call option on future investment, i.e., $V + \max(xV - I_E, 0)$. Given an initial design choice, management may deliberately favor a more expensive technology for the built-in flexibility to expand production if and when it becomes desirable. As discussed further below, the option to expand may also be of strategic importance, especially if it enables the firm to capitalize on future growth opportunities. As noted, when the firm buys vacant undeveloped land, or when it builds a small plant in a new geographic location (domestic or overseas) to position itself to take advantage of a developing large market, it essentially installs an expansion/growth option. This option, which will be exercised only if future market developments turn out favorable, can make a seemingly unprofitable (based on static NPV) base-case investment worth undertaking.

(*iv*) *The option to contract.* If market conditions are weaker than originally expected, management can operate below capacity or even reduce the scale of operations (by $c\%$), thereby saving part of the planned investment outlays (I_C). This flexibility to mitigate loss is analogous to a put option on part ($c\%$) of the base-scale project, with exercise price equal to the potential cost savings (I_C), giving $\max(I_C - cV, 0)$. The option to contract, just as the option to expand, may be particularly valuable in the case of new product introductions in uncertain markets. The option to contract may also be important, for example, in choosing among technologies or plants with a different construction to maintenance cost mix, where it may be preferable to build a plant with lower initial construction costs and higher maintenance expenditures in order to acquire the flexibility to contract operations by cutting down on maintenance if market conditions turn out unfavorable.

(*v*) *The option to shut down (and restart) operations.* In real life, the plant does not have to operate (i.e., extract oil) in each and every period automatically. In fact, if oil prices are such that cash revenues are not sufficient to cover variable operating (e.g., maintenance) costs, it might be better not to operate temporarily, especially if the costs of switching between the operating and idle modes are relatively small. If prices rise sufficiently, operations can start again. Thus, operation in each year can be seen as a call option to acquire that year's cash revenues (C) by paying the variable costs of operating (I_V) as exercise price, i.e., $\max(C - I_V, 0)$.[2] Options to alter the operating scale (i.e., expand, contract, or shut down) are typically found in natural resource industries, such as mine operations, facilities planning and construction in cyclical industries, fashion apparel, consumer goods, and commercial real estate.

(*vi*) *The option to abandon for salvage value.* If oil prices suffer a sustainable decline or the operation does poorly for some other reason, management does not have to continue incurring the fixed costs. Instead, management may have a valuable option

[2]Alternatively, management has an option to obtain project value V (net of fixed costs, I_F) minus variable costs (I_V), or shut down and receive project value minus that year's foregone cash revenue (C), i.e., $\max(V - I_V, V - C) - I_F = (V - I_F) - \min(I_V, C)$. The latter expression implies that the option not to operate enables management to acquire project value (net of fixed costs) by paying the minimum of variable costs (if the project does well and management decides to operate) or the cash revenues (that would be sacrificed if the project does poorly and it chooses not to operate).

to abandon the project permanently in exchange for its salvage value (i.e., the resale value of its capital equipment and other assets in secondhand markets). As noted, this option can be valued as an American put option on current project value (V) with exercise price the salvage or best alternative use value (A), entitling management to receive $V + \max(A - V, 0)$ or $\max(V, A)$. Naturally, more general-purpose capital assets would have a higher salvage and option abandonment value than special-purpose assets. Valuable abandonment options are generally found in capital intensive industries, such as in airlines and railroads, in financial services, as well as in new product introductions in uncertain markets.

(*vii*) *The option to switch use (i.e., inputs or outputs).* Suppose the associated oil refinery operation can be designed to use alternative forms of energy inputs (e.g., fuel oil, gas, or electricity) to convert crude oil into a variety of output products (e.g., gasoline, lubricants, or polyester). This would provide valuable built-in flexibility to switch from the current input to the cheapest future input, or from the current output to the most profitable future product mix, as the relative prices of the inputs or outputs fluctuate over time. In fact, the firm should be willing to pay a certain positive premium for such a flexible technology over a rigid alternative that confers no choice or less choice. Indeed, if the firm can in this way develop extra uses for its assets over its competitors, it may be at a significant advantage. Generally, "process" flexibility can be achieved not only via technology (e.g., by building a flexible facility that can switch among alternative energy "inputs"), but also by maintaining relationships with a variety of suppliers, changing the mix as their relative prices change. Subcontracting policies may allow further flexibility to contract the scale of future operations at a low cost in case of unfavorable market developments. As noted, a multinational oil company may similarly locate production facilities in various countries in order to acquire the flexibility to shift production to the lowest-cost producing facilities, as the relative costs, other local market conditions, or exchange rates change over time. Process flexibility is valuable in feedstock-dependent facilities, such as oil, electric power, chemicals, and crop switching. "Product" flexibility, enabling the firm to switch among alternative "outputs," is more valuable in industries such as automobiles, consumer electronics, toys or pharmaceuticals, where product differentiation and diversity are important and/or product demand is volatile. In such cases, it may be worthwhile to install a more costly flexible capacity to acquire the ability to alter product mix or production scale in response to changing market demands.

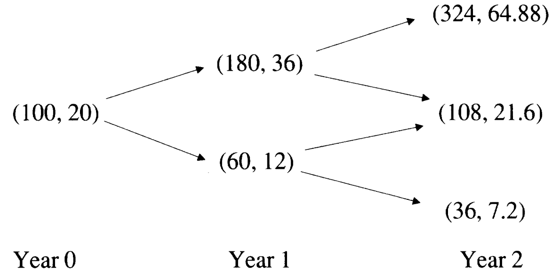
(*viii*) *Corporate growth options.* As noted, another version of the earlier option to expand of considerable strategic importance are corporate growth options that set the path of future opportunities. Suppose, in the above example, that the proposed refinery facility is based on a new, technologically superior "process" for oil refinement developed and tested internally on a pilot plant basis. Although the proposed facility in isolation may appear unattractive, it could be only the first in a series of similar facilities if the process is successfully developed and commercialized, and may even lead to entirely new oil by-products. More generally, many early investments (e.g., R&D, a lease on undeveloped land or a tract with potential oil reserves, a strategic acquisition, or an information technology network) can be seen as prerequisites or links in a chain of interrelated projects. The value of these projects may derive not so much from their expected directly measurable cash flows, but rather from unlocking future growth opportunities (e.g., a new-generation product or process, oil reserves, access to a new or expanding market, strengthening of the firm's core capabilities or strategic positioning). An opportunity to invest in a first-generation high-tech product, for example, is analogous to an option on options (an interproject compound option). Despite a seemingly negative NPV, the infrastructure, experience, and potential by-products generated during the development of the first-generation product may serve as springboards for developing lower-cost or improved-quality future generations of that product, or even for generating new applications into other areas. But unless the firm makes that initial investment, subsequent generations or other applications would not even be feasible. The infrastructure and experience gained can be proprietary and can place the firm at a competitive advantage, which may even reinforce itself if learning cost curve effects are present. Growth options are found in all infrastructure-based or strategic industries, especially in high-tech, R&D, or industries with multiple product generations or applications (e.g., semiconductors, computers, pharmaceuticals), in multinational operations, and in strategic acquisitions.

In a more general context, such operating and strategic adaptability represented by corporate real options can be achieved at various stages during the value chain, from switching the factor input mix among various suppliers and subcontracting practices, to rapid product design (e.g., computer-aided design) and modularity in design, to shifting production among various products rapidly and cost-efficiently in a flexible manufacturing system. The next section illustrates, through simple numerical examples, basic practical principles for valuing several of the above real options. For expositional simplicity, we will subsequently ignore any return shortfall or other dividend-like effects (see Section I.D. above for appropriate adjustments).

B. Principles of Valuing Various Real Options

Consider, as in Trigeorgis and Mason [110],[3] valuing a generic investment opportunity (e.g., similar to the above oil extraction project). Specifically, suppose we are faced with an opportunity to invest $I_0 = \$104$ (in millions) in an oil project whose (gross) value in each period will either move up by 80% or down by 40%, depending on oil price fluctuations: a year later, the project will have an expected value (from subsequent cash flows) of \$180 (million) if the oil price moves up ($C^+ = 180$) or \$60 if it moves down ($C^- = 60$).[4] There is an equal probability ($q = 0.5$) that the price of oil will move up or down in any year. Let S be the price of oil, or generally of a "twin security" that is traded in the financial markets and has the same risk characteristics (i.e., is perfectly correlated) with the real project under consideration (such as the stock price of a similar operating unlevered oil company). Both the project and its twin security (or oil prices) have an expected rate of return (or discount rate) of $k = 20\%$, while the risk-free interest rate is $r = 8\%$.

In what follows, we assume throughout that the value of the project (i.e., the value, in millions of dollars, in each year, t, of its subsequent expected cash flows appropriately discounted back to that year), V_t, and its twin security price (e.g., a twin oil stock price in \$ per share, or simply the price of oil in \$ per barrel), S_t, move through time as follows:



For example, the pair (V_0, S_0) above represents a current gross project value of \$100 million, and a spot oil price of \$20 a barrel (or a \$20 per share twin oil stock price). Under traditional (passive) NPV analysis, the current gross project value would be obtained first by discounting the project's end-of-period values (derived from subsequent cash flows), using the expected rate of return of the project's twin security (or, here, of oil prices) as the appropriate discount rate, i.e., $V_0 = (0.5 \times 180 + 0.5 \times 60)/1.20 = 100$. Note that this gross project value is, in this case, exactly proportional to the twin security price (or the spot oil price). After subtracting the current investment costs, $I_0 = 104$, the project's NPV is finally given by:

$$NPV = V_0 - I_0 = 100 - 104 = -4\ (< 0). \tag{2}$$

In the absence of managerial flexibility or real options, traditional DCF analysis would have rejected this project based on its negative NPV. However, passive DCF is unable to properly capture the value of embedded options because of their discretionary asymmetric nature and dependence on future events that are uncertain at the time of the initial decision. The fundamental problem, of course, lies in the valuation of investment opportunities whose claims are not symmetric or proportional and whose discount rates vary in a complex way over time.

Nevertheless, such real options can be properly valued using contingent claims analysis (CCA) within a backward risk-neutral valuation process.[5] Essentially, the same solu-

[3]Trigeorgis and Mason [110] use a similar example to show how options-based valuation can be seen operationally as a special, though economically corrected, version of decision tree analysis (DTA) that recognizes open-market opportunities to trade and borrow.

[4]All project values are hereafter assumed to be in millions of dollars (with "millions" subsequently dropped).

[5]As noted, the basic idea is that management can replicate the payoff to equity by purchasing a specified number of shares of the "twin security" and financing the purchase in part by borrowing a specific amount at the

tion can be obtained in our actual risk-averse world as in a "risk-neutral" world in which the current value of any contingent claim could be obtained from its expected future values — with expectations taken over the risk-neutral probabilities, p, imputed from the twin security's (or oil) prices — discounted at the riskless rate, r. In such a risk-neutral world, the current (beginning of the period) value of the project (or of equityholders' claim), E, is given by:

$$E = \frac{pE^+ + (1-p)E^-}{(1+r)},$$

where

$$p = \frac{(1+r)S - S^-}{(S^+ - S^-)}. \tag{3}$$

The probability, p, can be estimated from the price dynamics of the twin security (or of oil prices):

$$p = \frac{(1.08 \times 20) - 12}{36 - 12} = 0.4.$$

Note that the value for $p = 0.4$ is distinct from the actual probability, $q = 0.5$, and can be used to determine "certainty-equivalent" values (or expected cash flows) which can be properly discounted at the risk-free rate. For example,

$$V_0 = \frac{pC^+ + (1-p)C^-}{(1+r)} = \frac{0.4 \times 180 + 0.6 \times 60}{1.08} = 100. \qquad (4)^6$$

In what follows, we assume that if any part of the required investment outlay (having present value of \$104 million) is not going to be spent immediately but in future installments, that amount is placed in an escrow account earning the riskless interest rate.[7] We next illustrate how various kinds of both upside-potential options, such as to defer or expand, and downside-protection options, such as to abandon for salvage or default during construction, can enhance the value of the opportunity to invest (i.e., the value of equity or NPV) in the above generic project, under the standard assumption of all-equity financing. Our focus here is on basic practical principles for valuing one kind of operating option at a time.

1. The Option to Defer Investment

The company has a one-year lease providing it a proprietary right to defer undertaking the project (i.e., extracting the oil) for a year, thus benefiting from the resolution of uncertainty about oil prices over this period. Although undertaking the project immediately has a negative NPV (of − 4), the opportunity to invest afforded by the lease has a positive worth since management would invest *only* if oil prices and project value rise sufficiently, while it has no obligation to invest under unfavorable developments. Since the option to wait is analogous to a call option on project value, V, with an exercise price equal to the required outlay next year, $I_1 = 112.32$ ($= 1.04 \times 1.08$):

$$E^+ = \max(V^+ - I_1, 0) = \max(180 - 112.32, 0) = 67.68,$$
$$E^- = \max(V^- - I_1, 0) = \max(60 - 112.32, 0) = 0. \tag{5}$$

The project's total value (i.e., the expanded NPV that includes the value of the option to defer) from Equation (3) is:

$$E_0 = \frac{pE^+ + (1-p)E^-}{(1+r)} = \frac{0.4 \times 67.68 + 0.6 \times 0}{1.08} = 25.07. \tag{6}$$

From Equation (1), the value of the option to defer provided by the lease itself is thus given by:

$$\text{Option to defer} = \text{expanded NPV} - \text{passive NPV} = 25.07 - (-4) = 29.07 \tag{7}$$

which, incidentally, is equal to almost one-third of the project's gross value.[8]

riskless interest rate, r. This ability to construct a "synthetic" claim or an equivalent/replicating portfolio (from the "twin security" and riskless bonds) based on no-arbitrage equilibrium principles enables the solution for the current value of the equity claim to be independent of the actual probabilities (in this case, 0.5) or investors' risk attitudes (the twin security's expected rate of return or discount rate, $k = 0.20$).

[6]This confirms the gross project value, $V_0 = 100$, obtained earlier using traditional DCF with the actual probability ($q = 0.5$) and the risk-adjusted discount rate ($k = 0.20$).

[7]This assumption is intended to make the analysis somewhat more realistic and invariant to the cost structure make-up, and is not at all crucial to the analysis.

[8]The above example confirms that CCA is operationally identical to decision tree analysis (DTA), with the key difference that the probabilities are transformed so as to allow the use of a risk-free discount rate. Note, however, that the DCF/DTA value of waiting may differ from that given by CCA. The DCF/DTA approach in this case will overestimate the value of the option if it discounts at the constant 20% rate required of securities comparable in risk to the "naked" (passive) project:

$$E_0 = \frac{qE^+ + (1-q)E^-}{(1+k)} = \frac{0.5 \times 67.68 + 0.5 \times 0}{1.20} = 28.20.$$

2. The Option to Expand (Growth Option)

Once the project is undertaken, any necessary infrastructure is completed and the plant is operating, management may have the option to accelerate the rate or expand the scale of production by, say, 50% ($x = 0.50$) by incurring a follow-up investment outlay of $I_E = 40$, provided oil prices and general market conditions turn out better than originally expected. Thus, in year 1 management can choose either to maintain the base scale operation (i.e., receive project value, V, at no extra cost) or expand by 50% the scale and project value by incurring the extra outlay. That is, the original investment opportunity is seen as the initial-scale project plus a call option on a future opportunity, or $E = V + \max(xV - I_E, 0) = \max(V, (1+x)V - I_E)$:

$$E^+ = \max(V^+, 1.5V^+ - I_E) = \max(180, 270 - 40) = 230$$

i.e., expand;

$$E^- = \max(V^-, 1.5V^- - I_E) = \max(60, 90 - 40) = 60 \qquad (8)$$

i.e., maintain base scale. The value of the investment opportunity (including the value of the option to expand if market conditions turn out better than expected) then becomes:

$$E_0 = \frac{pE^+ + (1-p)E^-}{(1+r)} - I_0 = \frac{0.4 \times 230 + 0.6 \times 60}{1.08} - 104 = 14.5, \qquad (9)$$

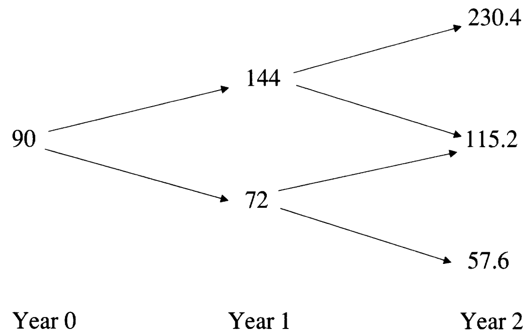
and thus the value of the option to expand is:

$$\text{Option to expand} = 14.5 - (-4) = 18.5, \qquad (10)$$

or 18.5% of the gross project value.

3. Options to Abandon for Salvage Value or Switch Use

In terms of downside protection, management has the option to abandon the oil extraction project at any time in exchange for its salvage value or value in its best alternative use, if oil prices suffer a sustainable decline. The associated oil refinery plant also can use alternative energy inputs and has the flexibility to convert crude oil into a variety of products. As market conditions change and the relative prices of inputs, outputs or the plant resale value in a secondhand market fluctuate, equityholders may find it preferable to abandon the current project's use by switching to a cheaper input, a more profitable output, or simply sell the plant's assets to the secondhand market. Let the project's value in its best alternative use, A, (or the salvage value for which it can be exchanged) fluctuate over time as:



Note that the project's current salvage or alternative use value ($A_0 = 90$) is below the project's value in its present use ($V_0 = 100$) — otherwise management would have switched use immediately — and has the same expected rate of return (20%); it nevertheless has a smaller variance so that if the market keeps moving up it would not be optimal to abandon the project early for its salvage value, but if it moves down management may find it desirable to switch use (e.g., in year 1 exchange the present use value of $V_1 = 60$ for a higher alternative use value of $A_1 = 72$).[9] Thus, equityholders can choose the maximum of the project's value in its present use, V, or its value in the best alternative use, A, i.e., $E = \max(V, A)$:

$$E^+ = \max(V^+, A^+) = \max(180, 144) = 180 = V^+,$$

i.e., continue;

$$E^- = \max(V^-, A^-) = \max(60, 72) = 72 = A^-, \qquad (11)$$

i.e., switch use. The value of the investment (including the option to abandon early or switch use) is then:

Again, the error in the traditional DTA approach arises from the use of a single (or constant) risk-adjusted discount rate. Asymmetric claims on an asset do not have the same riskiness (and hence expected rate of return) as the underlying asset itself. CCA corrects for this error by transforming the probabilities.

[9]We assume here for simplicity that the project's value in its current use and in its best alternative use (or salvage value) are perfectly positively correlated. Of course, the option to switch use would be even more valuable the lower the correlation between V and A.

$$E_0 = \frac{pE^+ + (1-p)E^-}{(1+r)} - I_0$$
$$= \frac{0.4 \times 180 + 0.6 \times 72}{1.08} - 104 = +2.67 \qquad (12)$$

so that the project with the option to switch use is now desirable. The value of the option itself is:

$$\text{Option to switch use} = 2.67 - (-4) = 6.67, \qquad (13)$$

or almost seven percent of the project's gross value. This value is clearly dependent on the schedule of salvage or alternative use values.

4. The Option to Default (on Planned Staged Cost Installments) During Construction

Even during the construction phase, management may abandon a project to save any subsequent investment outlays, if the coming required investment exceeds the value from continuing the project (including any future options). Suppose that the investment (of \$104 present value) necessary to implement the oil extraction project can be staged as a series of "installments": I_0 = \$44 out of the \$104 allocated amount will need to be paid out immediately (in year 0) as a start-up cost for infrastructure, with the \$60 balance placed in an escrow account (earning the risk-free rate) planned to be paid as a I_1 = \$64.8 follow-up outlay for constructing the processing plant in year 1. Next year management will then pay the investment cost "installment" as planned only in return for a higher project value from continuing, else it will forego the investment and receive nothing. Thus, the option to default when investment is staged sequentially during construction translates into $E = \max(V - I_1, 0)$:

$$E^+ = \max(V^+ - I_1, 0) = \max(180 - 64.8, 0) = 115.2,$$

i.e., continue;

$$E^- = \max(V^- - I_1, 0) = \max(60 - 64.8, 0) = 0, \qquad (14)$$

i.e., default. The value of the investment opportunity (with the option to default on future outlays) is given by:

$$E_0 = \frac{pE^+ + (1-p)E^-}{(1+r)} - I_0$$
$$= \frac{0.4 \times 115.2 + 0.6 \times 0}{1.08} - 44 = -1.33 \qquad (15)$$

and the option to abandon by defaulting during construction is:

$$\text{Option to abandon by defaulting} = -1.33 - (-4) = 2.67, \qquad (16)$$

or about three percent of project value. This value is of course dependent on the staged cost schedule.

For simplicity, the above examples were based on a one-period risk-neutral backward valuation procedure. This procedure can be easily extended to a discrete multi-period setting with any number of stages. Starting from the terminal values, the process would move backwards calculating option values one step earlier (using the up and down values obtained in the preceding step), and so on. A two-period extension is illustrated in the next section. As the number of steps increases, the discrete-time solution naturally approaches its continuous Black-Scholes-type equivalent (with appropriate adjustments), when it exists.

In the next section, we turn to various financial flexibility options, starting with equityholders' option to default on debt payments deriving from limited liability. A similar financial abandonment option held by the lender can be created through staged financing. Interactions among such financial flexibility and the earlier operating options are explored.

III. Interactions With Financial Flexibility

A. Equityholders' Option to Default on Debt (Limited Liability)

So far we have dealt with various operating or real options, implicitly assuming an all-equity firm. If we allow for debt financing, then the value of the project to equityholders can potentially improve by the additional amount of financial flexibility (or the option to default on debt payments deriving from limited liability) beyond what is already reflected in the promised interest rate. We can illustrate how to incorporate the value of financial flexibility by reevaluating the original investment opportunity with project financing (where the firm consists entirely of this oil project). Consider, for example, venture capital financing of a single-project start-up oil company. Suppose initially that venture capitalists (or "junk" bond purchasers) would be content to provide funds in exchange for contractually promised fixed-debt payments, and require an equilibrium return on comparably risky bonds (that already reflects a premium for equity's option to default) of 16.7%.[10, 11]

Specifically, suppose that $I_0^D = \$44$ out of the required immediate \$104 outlay is borrowed against the investment's expected future cash flows to be repaid with interest in two years at the promised equilibrium interest rate of 16.7% per year. The balance of $I_0^E = \$60$ is supplied by the firm's equityholders (i.e., the entrepreneurs). Equityholders, of course, have an option to acquire the firm (project) value V — which in the meantime is "owned" by the debtholders (here, the venture capitalists) — by paying back the debt (with imputed interest) as exercise price two years later. Thus, in year 2, equityholders will pay back what they owe the debtholders ($D_2 = 44 \times 1.167^2 = 59.92$) only if the investment value exceeds the promised payment, else they will exercise their limited liability rights to default (i.e., surrender the project's assets to debtholders and receive nothing), or $E_2 = \max(V_2 - D_2, 0)$. Thus, depending on whether oil prices move up in both years (++), up in one year and down in the other (+ –) or down in both years (– –), the equityholders' claims in year 2 will be:

$$E_2^{++} = \max(324 - 59.92, 0) = 264.08,$$

$$E_2^{+-} = E_2^{-+} = \max(108 - 59.92, 0) = 48.08,$$

$$E_2^{--} = \max(36 - 59.92, 0) = 0.$$

The value of equityholders' claims back in year 1, depending on whether the oil market was up or down, would then be, according to CCA:

$$E_1^+ = \frac{pE_2^{++} + (1-p)E_2^{+-}}{(1+r)} = \frac{0.4 \times 264.08 + 0.6 \times 48.08}{1.08} = 124.52,$$

$$E_1^- = \frac{pE_2^{-+} + (1-p)E_2^{--}}{(1+r)} = \frac{0.4 \times 48.08 + 0.6 \times 0}{1.08} = 17.81.$$

Finally, moving another step back to year 0, the present value of the oil investment opportunity (with partial debt financing) is:

$$E_0 = \frac{pE_1^+ + (1-p)E_1^-}{(1+r)} - I_0^E = \frac{0.4 \times 124.52 + 0.6 \times 17.81}{1.08} - 60 = -4. \quad (17)$$

This (expanded or adjusted NPV) value is the same as the NPV of the all-equity financed project found in Equation (2), confirming that debt financing at the 16.7% equilibrium interest rate (that already reflects a premium for the equityholders' option to default) is a zero-NPV transaction.[12] Since, in this case, the promised 16.7% interest

[10]For a good qualitative discussion of venture capital financing arrangements, see Sahlman [88]. Mauer and Triantis [72] present another treatment of dynamic interactions between corporate financing and investment decisions, where they refer to financial flexibility as the ability to adjust the firm's debt level over time (recapitalization).

[11]In addition to contractually fixed debt (or preferred stock) payments (at a high required rate), venture capitalists may want part of their compensation in the form of a percentage ownership of the equity of the firm (or in the form of warrants). Some venture capitalists (especially in an LBO context), however, may prefer to place their funds in the form of debt rather than common equity since they can generally exercise more effective control over their investment through the debt's covenants than through the stock's voting power. The debt principal may also provide a better mechanism for a tax-free recovery of capital for young privately held firms that may not be feasible with stock until the company goes public. Initially we consider here the simpler case of all-debt venture capital financing, but later consider mixed debt-equity financing by venture capitalists.

[12]The 16.7% equilibrium return demanded by lenders that takes the firm's option to default into account in pricing the debt can be determined as the promised debt interest rate (r_D) derived from the difference between the face value of the debt to be repaid at the end of the two periods (B) and the current value of the debt ($D_0 \equiv I_0^D = \$44$). The debt face value, B, is the amount that satisfies the condition that the discounted expected terminal payoff to the debtholders in each state i (D_2^i) under risk-neutral valuation equals the current debt amount, i.e., $\sum p^i D_2^i/(1+r)^2 = 44$, where the debtholders' terminal payoff is the minimum of the face value of the debt or the value of the firm at default, $D_2^i = \min(B, V_2^i)$. In the above example, at terminal period 2:

$$D_2^{++} = \min(B, 324) = B,$$

$$D_2^{+-} = D_2^{-+} = \min(B, 108) = B,$$

$$D_2^{--} = \min(B, 36) = 36.$$

The value of debtholders' claims back in year 1 then is:

$$D_1^+ = \frac{pD_2^{++} + (1-p)D_2^{+-}}{(1+r)} = \frac{0.4B + 0.6B}{1.08} = \frac{B}{1.08},$$

$$D_1^- = \frac{pD_2^{-+} + (1-p)D_2^{--}}{(1+r)} = \frac{0.4B + 0.6 \times 36}{1.08}.$$

Finally, moving another step back to year 0:

$$D_0 = \frac{pD_1^+ + (1-p)D_1^-}{(1+r)},$$

or

$$44 = \frac{0.4B + 0.6(0.4B + 21.6)}{1.08^2},$$

resulting in $B = 59.94$. From $D_0(1 + r_D)^2 = B$ with $D_0 = 44$, this implies that $r_D = 16.7\%$. The fact that the project NPV remains unchanged with debt financing in Equation (17) confirms that this is the equilibrium rate that fairly prices the default option ex ante.

rate on debt is an equilibrium return, the project's NPV does not change with the introduction of debt financing. The firm compensates the lenders ex ante through a fair default option premium embedded in the promised equilibrium rate in exchange for financial flexibility.

Of course, if lenders were to accept a lower promised interest rate of, say, 12% that did not incorporate fully a fair premium for the option to default, E_0 above would instead be -1.40, resulting in an additional value of financial flexibility to equityholders (resulting from the option to default on debt) of $-1.40 - (-4) = 2.60$, or about three percent of the investment's gross value. In such a case, potential interactive effects between operating and financial flexibility may further magnify the amount of undervaluation caused by traditional DCF techniques. We next consider the presence of both financial flexibility (deriving from equityholders' limited liability rights to default) and the operating default option analyzed earlier.

B. Potential Interaction Between Operating and Financial Default Flexibilities

Suppose now that I_0^D = \$44 were borrowed as before from venture capital sources (or by issuing junk bonds) to be used immediately as an investment start-up cost for infrastructure, while the \$60 equity contribution is to be potentially expended (with earned interest) as a second-stage investment "installment" for building the processing plant in year 1 (as $I_1^E = 64.8$).[13] Thus, equityholders now have extra operating flexibility to abandon the project (by choosing not to expend the "equity cost installment," I_1^E, if it turns out to exceed the project's value) in year 1.

Again, starting from the end and moving backward, the value of equity's claims in year 2 (with debt repayment) remains unchanged, but in year 1 now becomes the maximum of its value in the previous case (in the absence of any outlay for continuing) minus the "equity cost" I_1^E now due, or zero (if the project performs poorly and equityholders default), i.e., $(E_1)' = \max(E_1 - I_1^E, 0)$:

$$(E_1^+)' = \max(124.52 - 64.8, 0) = 59.72 \text{ (continue);}$$

$$(E_1^-)' = \max(17.81 - 64.8, 0) = 0 \text{ (abandon).}$$

The value of the investment (with both operating and financial default flexibility) is:

$$E_0{}' = \frac{p(E_1^+)' + (1-p)(E_1^-)'}{(1+r)} = \frac{0.4 \times 59.72 + 0.6 \times 0}{1.08} = 22.12. \quad (18)$$

Thus, the incremental value of the operating default option in the presence of financial flexibility is $22.12 - (-4) = 26.12$ or about one-fourth of gross investment value, far exceeding the three percent value of the equivalent operating option to default under all-equity financing in Equation (16) above. This confirms that the incremental value of an option in the presence of other options may differ significantly from its individual value in isolation, and that financial and operating flexibility options may interact. These option interactions may be more pronounced if lenders accept a lower interest than the fair equilibrium return of 16.7%. For example, had the promised interest rate been only 12%, $E_0{}'$ would instead be 23.74 and the combined value of the operating option to default on planned cost installments (determined separately to be about three percent in Equation (16)) with the extra financial flexibility to default on debt (separately estimated at about three percent in the preceding section) would be about 28%. This combined value far exceeds the sum of separate option values, indicating the presence of substantial positive interaction (i.e., 28% > (3 + 3)%). Such positive interaction effects are typical in compound option situations such as these.[14]

C. Venture Capitalists' (Lender's) Option to Abandon Via Staged Debt Financing

So far we have focused on the financial option to default on debt payments held by the equityholders (entrepreneurs). The venture capitalists, however, may also wish to generate an option to abandon the venture themselves by insisting on providing staged or sequential capital financing. For example, they could insist on actually providing only half the requested \$44 amount up front, I_0^D = \$22 (to be repaid at the 16.7% required rate as \$29.96 in two years), with the remaining portion (allowed to grow at the eight percent riskless interest rate, $I_1^D = \$22 \times 1.08 = 23.76$) to be provided next year, contingent on successful interim progress. Following a successful first stage, the second stage would be less risky so that a lower 12% rate would be agreeable (with the \$23.76 to return \$26.61 a year later). The equityholders would thus also need to contribute ($I_0^E = 22$) toward the \$44 upfront cost for infrastructure ($I_0 = I_0^D + I_0^E = 22 + 22$), as well as ($I_1^E = 41.04$) toward the

[13]Notice that this case is identical to the operating default case in Section II.B.4. above, with the only difference being that the initial outlay now comes from borrowed money.

[14]See also Trigeorgis [106] for the nature of real option interactions.

potential second-stage \$64.8 processing plant cost one year later ($I_1 = I_1^D + I_1^E = 23.76 + 41.04$), if the venture at that time appears worth pursuing further.

Suppose that the venture capitalists would choose to provide second-stage financing (at the lower 12% rate) *only* if the first stage is successful (i.e., following a "+" oil price state in period 1), but would otherwise choose to abandon the venture in midstream. In this case, equityholders' value in the intermediate states in year 2 may differ, contingent on first year apparent success. That is, E_2^{+-} would differ from E_2^{-+}, since, in the first case, the venture capitalists would be repaid \$26.61 for the second-stage financing they would provide following a successful first stage, in addition to the \$29.96 repayment for the upfront debt financing. Thus,

$$E_2^{++} = \max(324 - (29.96 + 26.61), 0) = 267.43$$

$$E_2^{+-} = \max(108 - 56.57, 0) = 51.43$$

(while following a "–" state in period 1 only the upfront debt repayment need be made:

$$E_2^{-+} = \max(108 - 29.96, 0) = 78.04$$

$$E_2^{--} = \max(36 - 29.96, 0) = 6.04.)$$

If there were no outlays required in period 1, the value of equityholders' claims would be:

$$E_1^+ = \frac{0.4 \times 267.43 + 0.6 \times 51.43}{1.08} = 127.62$$

$$\text{(with } E_1^- = \frac{0.4 \times 78.04 + 0.6 \times 6.04}{1.08} = 32.26).$$

Since equityholders would actually need to contribute $I_1^E = 41.04$ in period 1 for the venture to proceed, the correct (revised) value is the maximum of the above value in the absence of any outlays minus the "equity cost" I_1^E, or zero (if the venture performs poorly and is abandoned in midstream), i.e., $(E_1)' = \max(E_1 - I_1^E, 0)$:

$$(E_1^+)' = \max(127.62 - 41.04, 0) = 86.58,$$

but when $(E_1^-)' = 0$, after a disappointing first stage, the venture would be abandoned. Finally, the time-0 value of equityholders' claims becomes:

$$E_0' = \frac{p(E_1^+)' + (1-p)(E_1^-)'}{(1+r)} - I_0^E = \frac{0.4 \times 86.58 + 0.6 \times 0}{1.08} - 22 = 10.07. \quad (19)$$

Thus, the value of equity's default options, offset by the venture capitalists' option to abandon by refusing to provide second-stage financing, is 10.07 – (– 4) = 14.07 or 14% of gross project value.

This value is less than the 26% equity default option value found in Subsection B above, without the venture capitalists' abandonment option. The venture capitalists should thus be willing to pay a premium of up to \$12 (million) to preserve their option to abandon via staged debt financing. Still, the above value (14) is in excess of that in Section III.A., where the full \$44 borrowed amount was unequivocally committed upfront. In the present case, venture capitalists are better off via their option to abandon the venture by refusing to contribute second-stage financing in case of interim failure. This, in turn, enables the equityholders to obtain better financing terms, such as saving on debt interest costs.

Indeed, as discussed further below, structuring the financing deal in contingent stages to more closely match the inherent resolution of uncertainty over the investment's different stages can make both parties better off. For example, providing equity financing in stages, rather than all upfront, would not only benefit the venture capitalists via their option to abandon, but may also allow the entrepreneurs to raise equity capital later at a potentially more favorable valuation resulting in less equity dilution. Even following a bad interim state, entrepreneurs (who presumably have more information and may still believe the project is worthwhile to pursue) can prevent abandonment of the venture by the lenders by renegotiating more appropriate second-stage financing terms given the revealed higher risks, thus generating mutual gains by solving the underlying agency or underinvestment problem in this case. More generally, the flexibility to actively revalue the terms of a financing deal to better match the evolution of operating project risks, whether increasing or decreasing, as the project moves into its various stages creates value, compared to a passive alternative where the financing terms are irrevocably committed to from the outset under less complete information. The value created by partially solving this information problem via flexible, contingent financing arrangements can be of mutual benefit to both parties.

D. Mixed (Debt-Equity) Venture Capital Financing

Consider now the case where the venture capitalists finance the full \$44 start-up cost, half in the form of debt (to be repaid at a 16.7% rate as \$29.96 in two years) and the other half in exchange for an upfront 22% equity ownership share.[15] Thus, both the total equity expected return and the risk are divided proportionately (78/22%) among the entrepreneurs and the venture capitalists. The group of equityholders would still make an upfront contribution of $I_0^E = 22$ (using the cash provided by venture capitalists in exchange for the equity share), and may incur a discretionary follow-up equity cost outlay of $I_1^E = 64.8$ if the project proceeds well. In this case,

$$E_2^{++} = \max(324 - 29.96, 0) = 294.04,$$
$$E_2^{+-} = E_2^{-+} = \max(108 - 29.96, 0) = 78.04,$$
$$E_2^{--} = \max(36 - 29.96, 0) = 6.04.$$

In the absence of a period-1 outlay, the value of equityholders' claims in year 1 would be:

$$E_1^+ = \frac{0.4 \times 294.04 + 0.6 \times 78.04}{1.08} = 152.26,$$
$$E_1^- = \frac{0.4 \times 78.04 + 0.6 \times 6.04}{1.08} = 32.26.$$

Adjusting for the $I_1^E = 64.8$ discretionary outlay in case the project is continued,

$$(E_1^+)' = \max(152.26 - 64.8, 0) = 87.46$$

i.e., continue;

$$(E_1^-)' = \max(32.26 - 64.8, 0) = 0,$$

since equityholders would abandon the venture. Finally, the time-0 value of the combined equityholder group's claims (with default flexibility) is:

$$E_0' = \frac{0.4 \times 87.46 + 0.6 \times 0}{1.08} = 32.4. \qquad (20)$$

The entrepreneurs would receive 78% of this $32.4 net value, or $25.27 (million). This represents an improvement over the $22.12 value of an all-debt capital upfront commitment of Equation (18) (as well as compared to the $10.07 value in the previous case of all-debt staged financing of Equation (19), that gives venture capitalists an option to abandon). Note further that this case of mixed debt-equity financing results in a gross investment value (after adding the 104 costs) of $136.4. Of this total value, 22% or $30 would go to the venture capitalists (in return for their $22 initial equity investment). Venture capitalists are also better off in the case of staged debt financing (compared to an upfront capital commitment) since they would have better control of (part of) their funds, especially in the event of disappointing interim results.

If venture capital equity financing is also provided in stages, the reduced operating uncertainties (as the project proceeds into its later stages) and the higher value to the venture capitalists following a successful first stage can result in less equity dilution for the entrepreneurs. For example, suppose that the venture capitalists again provide the first $22 upfront in the form of debt, but postpone the decision to contribute the rest ($23.76 in a year) in exchange for an equity share to be determined contingent on successful interim progress next year. The year-2 equity values would remain the same as above, and in period 1 would change only to the extent that now $I_1^E = 41.04$ (since 23.76 of the 64.8 discretionary year-1 outlay will now be provided by venture capitalists in exchange for equity if the first stage is successful). Thus,

$$(E_1^+)'' = \max(152.26 - 41.04, 0) = 111.22 \text{ (continue)},$$
$$(E_1^-)'' = 0 \text{ (abandon)}.$$

If, contingent on first-stage success, venture capitalists can receive a 13.5% equity share in exchange for their $23.76 contribution, the entrepreneurs would then obtain 86.5% of $111.22 or $96.2 in the good state. Thus, the entrepreneurs' time-0 value would be:

$$E_0'' = \frac{0.4 \times 96.2 + 0.6 \times 0}{1.08} - 22 = 13.63. \qquad (21)$$

This exceeds the $10.07 value of Equation (19) obtained under all-debt staged financing, with the $3.56 difference representing savings due to the lower equity dilution as a result of the more flexible, contingent arrangement. Thus, staging equity financing sequentially would not only make the venture capitalists better off (by generating an option to abandon), but would also allow the entrepreneurs to raise equity capital later at a potentially more favorable valuation. These results confirm that both parties can be better off if the financing deal is flexibly arranged such that it better matches the evolution of operating project risks and valuation.

[15]Note that the $22 committed now amounts to 22% of the gross project value of $100, assuming a required 20% return on an equity position of comparable risk.

IV. Summary, Conclusions and Extensions

Following a comprehensive thematic overview of the evolution of real options, this paper has illustrated, through simple examples, how to quantify in principle the value of various types of operating options embedded in capital investments, both for enhancing upside potential (e.g., through options to defer or expand), as well as for reducing downside risk (e.g., via options to abandon for salvage value or switch use, and to default on staged planned outlays). We have also noted a number of fruitful future research directions, including more applications and implementation problems, empirical and field studies, theoretical extensions combining options theory with Bayesian analysis to model learning, with game theory to model competitive and strategic interactions, with agency theory/asymmetric information to model/correct misuse of managerial discretion, as well as interactions between operating and financial flexibility.

Taking a first step in the latter direction, we extended the analysis in the presence of leverage within a venture capital context and examined the potential improvement in equityholders' value as a result of additional financial flexibility, starting from the equityholders' option to default on debt payments deriving from limited liability. The beneficial impact of staging venture capital financing in installments, thereby creating an option to abandon by the lender, and when using a mix of debt and equity venture capital was also examined. Staging capital financing may be beneficial not only to venture capitalists (by preserving an option to abandon), but also to entrepreneurs as well, since it allows potentially better financing terms in later stages. In later-stage debt financing, for example, better terms may be achieved in the form of lower interest costs. If later-stage financing is to be provided in the form of an equity ownership share based on the project's market value as would be revealed at an interim stage, entrepreneurs could gain by suffering less equity dilution when a higher project value is assessed in reallocating the claims in the good interim state. Even in a bad interim state, entrepreneurs might still gain if they can prevent imminent abandonment of the venture (assuming they still believe it is worthwhile to pursue) by the venture capitalists by renegotiating more appropriate terms given the higher risks (either offering a greater equity share or a higher interest rate). The option to actively revalue the terms of a financing deal as operating project uncertainties get resolved over successive stages is clearly valuable, compared to a passive alternative where the financing terms are irrevocably committed to from the very beginning under less complete information. Building-in flexibility in a financing deal may determine whether the venture will continue and eventually succeed or fail when interim performance does not meet initial expectations.

Thus, contrary to what is often popularly assumed, the value of an investment deal may not depend solely on the amount, timing, and operating risk of its measurable cash flows. The future operating outcomes of a project can actually be impacted by future decisions (by either equityholders or lenders) depending on the inherent or built-in operating and financial options and the way the deal is financed (e.g., the staging of financing or the allocation of cash flows among debt and equity claimants). In such cases, interactions between a firm's operating and financial decisions can be quite significant, as exemplified by the typical venture capital case. These interactions are likely to be more pronounced for large, uncertain, long-development and multistaged investments or growth opportunities, especially when substantial external (particularly debt) multistaged financing is involved. Understanding these interactions and designing a proper financing deal that recognizes their true value, while being flexible enough to better reflect the evolution of a project's operating risks as it moves through different stages, can mean the difference between success or failure. Options-based valuation can thus be a particularly useful tool to corporate managers and strategists by providing a consistent and unified approach toward incorporating the value of both the real and financial options associated with the combined investment and financial decision of the firm.

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